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Heuristics for puzzle-based storage systems driven by a limited set of automated guided vehicles

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Abstract Storage systems play a vital role in industrial operations, and containment of their cost can be an important managerial issue. One way to achieve this aim is to increase storage density, i.e. to increase the area dedicated to storage and to reduce the size of aisles used to access goods. Innovative "puzzle-based" storage systems, in which shelves are moveable and hence allow the flexible arrangement of aisles, can be a promising solution, as long as retrieval times are fast enough and the required investment is not too high. The paper proposes the study of an innovative management solution for puzzle-based storage systems based on AGV tractors instead of self-propelled shelves. These systems have been studied by analyzing density and retrieval time as focal design variables.

Keywords Storage system design · Puzzle-based storage systems · AGV tractors · Handling strategies

Introduction

Storage systems play a vital role in industry at both production and distribution sites, to the point that some authors have claimed warehousing to be a strategic competency that companies can use to enhance their competitive position (Tompkins and Smith 1998). Even though contemporary approaches to operations management suggest to reduce the size of inventory, warehouses still are economically relevant not only because of the capital they lock up, but also because of cost and space associated to the warehouse itself (Briggs 1960). Costs depend on warehouse volume, equipment used and the management policies employed. For a survey on warehousing problems, the reader can refer to van den Berg and Zijm (1999).

In general, the size of the warehouse is positively correlated to costs, due to rental and energy associated to heating, lighting and transportation. Reducing the space dedicated to storage can therefore help contain the costs of warehousing. In some contexts, availability of space can also be a hard constraint. This may happen, for instance, in city centres (e.g., office archives, shops, car parks, etc.), on ships, etc. In these cases, the design of storage systems has to search for solutions that allow a high degree of efficiency in the use of surface and volume, consistent with the type of material to store, and the time needed to enter and retrieve items.

In a traditional storage system, goods are located on fixed shelves, while manned or automated vehicles move them to and from Input/Output (I/O) bays. Such systems necessarily include aisles in which the vehicles move, and these aisles take up space that could otherwise be used to store items. As pointed out by Yang and Kim (2006), when storage space is limited, it is necessary to develop sophisticated handling strategies and item relocation rules (i.e., how to relocate items when one of them is retrieved).

Puzzle-based storage systems try to avoid this problem by doing away with the concept of fixed aisles and vehicles. These systems belong to the class of automated dynamic storage systems (Bozer and White 1984; Meller and Mungwattana 1997), since items change their position while in the warehouse and goods handling does not require the use of workforce. Puzzle-based systems are inspired by the "15-sliding puzzle" that everyone knows as a children's game

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(Gardner 1959), in which 15 numbered tiles slide within a 4 × 4 grid. In a puzzle-based storage system, shelves slide along the two orthogonal directions, dynamically uncover the aisles, and are progressively shifted to load/unload bays that are usually located on the perimeter of the system. These storage systems can be designed in order to avoid structural changes on buildings and can be used in presence of pillars and partition walls; the shelves can be designed so that they slide against each other, therefore avoiding runners on the floor.

While a puzzle-based system can lead to higher system density, this comes at the cost of the time required for a given shelf to move through the system and reach the I/O point. Therefore, retrieval time becomes a key variable for system performance measurement, and shelf movements must be carefully planned in order to minimize it. This problem raises a number of interesting research opportunities.

The 15-sliding puzzle has been widely studied by scholars, and a number of efficient solution algorithms have been devised for it (a non-exhaustive list of such studies includes Parberry 1995; Ratner and Warmuth 1990; Kalermo 2000). However, there is a fundamental difference between the 15-sliding game and puzzle-based storage systems. While the objective of the game is to arrange tiles on the required positions, in the storage system one aims to make a single shelf (or a set of shelves, based on a picking list of items to warehoused) reach the load/unload location in the shortest time possible. To date, the only contribution dealing to this problem is the one proposed by Gue and Kim (2007) in their seminal work, which is based on the hypothesis that each shelf is independently propelled. These authors have proposed heuristics for managing shelf movements and have studied the inverse relationship between the number of empty slots in the system (termed "escorts") and the time needed to move a given shelf to the I/O bay. This relationship leads to a design trade-off between storage density and access time.

This paper proposes management strategies applicable to an alternative solution, in which a *limited* number of AGV tractors are dispatched to shelves and then provide for their movement. By not requiring a vehicle permanently attached to each shelf, this solution can lead to lower system cost, but it significantly complicates the management problem, at the same time introducing a further tradeoff, i.e., the one between access time and the number of tractors.

The paper is structured as follows. In the following section, puzzle-based systems based on limited numbers of AGV tractors are described in greater detail. Handling strategies for puzzle-based systems are discussed in "Shelf management strategies". "An application: car parking" reports the evaluation of these strategies and their comparison to traditional systems through simulation. The last section concludes the paper.

Puzzle-based storage systems using limited sets of AGV tractors

Physical description

In their paper, Gue and Kim discussed puzzle-based systems in which each shelf is a self-propelled platform. Especially in the case of warehouses with a significant number of shelves and/or dedicated to storing heavy items, this can be seen as an important drawback because of the investment required.

In order to reduce system cost, this paper proposes the use of sliding shelves that can be moved by Automated Guided Vehicles (AGVs). The reader can refer to Vis (2004) for a survey on AGVs and to Berman and Edan (2002) for the use of AGV in material handling systems. A further alternative that could be competitive with AGVs—but has not been considered in this paper—would be using automated lifting vehicles (ALVs) (Bae et al. 2009). The main elements of the innovative system proposed in the paper are shelves, robotized tractors (AGVs) and I/O points. Items are stored on shelves, and robotized tractors can be used to handle shelves and to transfer them to the I/O point by crawling under the shelf, securing a connection, and moving it in one of the two orthogonal directions.

The solution proposed can provide a number of benefits, beyond reducing the number of vehicles to be implemented. First, the control logic inside an AGV can be more sophisticated than in a self-propelled shelf, thus providing greater accuracy in directional guidance. This reduces the need to create a mechanically precise (but also complex) coupling between shelves, in order to ensure that elements are well-aligned to each other. Second, self-propelled shelves require energy to be distributed throughout the system, while AGV batteries can be recharged simply by taking them out of the system when necessary. Finally, it ensures a higher degree of reliability, since an AGV can readily be substituted by another one, while a broken self-propelled shelf can be difficult to access and could block the whole system.

Management strategies

The model proposed in literature to solve the management problem in a puzzle-based storage system is inspired by the movements required by the tiles of the 15-puzzle. In both cases, there are few open cells (a single one in the original game) and the tiles/shelves have to be moved without maneuvering them outside the grid/warehouse. The two problems are in fact quite similar: in order to move a tile or a shelf, an open location must be moved close to it and then the tile/shelf must be moved into the open location. In other words, the open location (called "escort") must precede the shelf on every step of its path, as showed in Fig. 1, where the black square is the shelf to be moved, the white one is the escort,

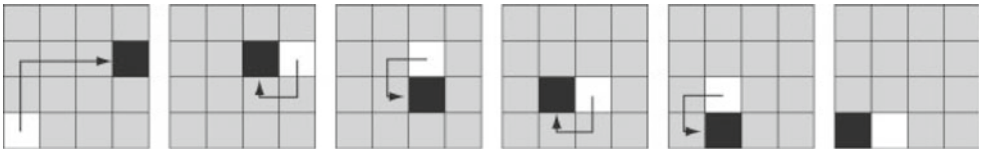


Fig. 1 The empty location "escorts" the requested shelf or tile to the desired location (the cell in lower-left corner)

and the arrow represents the moves. In practice, when one speaks about "moving an escort" from one point to an adjacent one, this is achieved with a U-shaped move of three shelves (referring to Fig. 1, the ones covered by the arrow, in the opposite direction to the arrow itself).

In warehousing applications, as already discussed, the problem to be solved is simpler than in the 15-puzzle, since the only issue at stake is the movement of the shelf to be picked, and not the arrangement of shelves in space. The problem is therefore also similar to the Rush Hour puzzle (Flake and Baum 2002). This other puzzle has the goal of "clearing the way" for the exit of a particular "vehicle" (i.e., identifying *clearing conditions*) by moving rectangular "vehicles" of different length that may move either vertically or horizontally, but not both. Puzzle-based warehouses are also simpler, because "vehicles" have the same unit size and may move in any direction.

In order to minimize retrieval time, the objective is to find a movement sequence that minimizes the number of shelves to handle; each movement consists in deciding which shelf to move and in which direction. Gue and Kim developed an optimal method of retrieving an item from puzzle-based systems with a single escort and developed a heuristic that produces optimal solutions for a large set of test problems for systems with multiple escorts. Their results show that, given a warehousing capacity, the number of escorts is negatively correlated to retrieval time. Exception made for very small systems, multiple-escort solutions are the only ones of practical use.

Within the context of this paper, where there are fewer tractors than shelves, there will be a further trade-off to be struck, between system cost and I/O time, which complements the previously discussed trade-off between I/O time and storage density. This additional trade-off makes it even more important to develop sound management strategies when planning shelf movements, since a better strategy can lead to the same performance, but at a lower system cost. At the same time, the management problem becomes more complicated, because it not only involves defining a sequence of items to be moved, but also which tractors to use and which paths to follow to reach the assigned shelf. In fact, tractor movements can have an important impact on system performance, because of the non-negligible time needed to reach the assigned shelf, hook onto it, rotate wheels to the desired direction make the shelf move, and unhook.

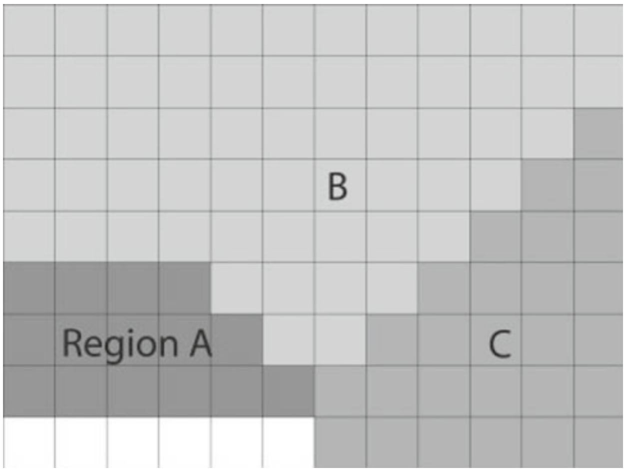


Fig. 2 Regions *A*, *B*, *C* as defined by Gue and Kim (2007)

Shelf management strategies

Autonomous-shelf strategies

Before discussing the management strategies applicable to the case of systems with a limited number of AGVs, it is appropriate to provide a short overview of Gue and Kim's approach, referring the reader to the original paper for details. The authors present a heuristics that seeks the minimum number of escort-moves to move an item in location (i, j) to the I/O point (1, 1), given an aisle of e escorts lined up at the I/O point. In the case of a multi-escort system, they identify three regions A, B, C (represented in Fig. 2) and apply a different rule for items located in each of the three regions.

In Region A, items are retrieved in an intuitive way, with interfering items moved down to the aisle and then right to its end, thus clearing an L-shaped path for the shelf to move directly to the I/O point (down and then left).

In Region B, the escort closest to the item is moved and is "puzzled" down and leftward, using distinct moves (defined as 3-moves and 5-moves) until the configuration meets one of four defined *clearing conditions*. As shown in Fig. 3a, a 3-move shifts the identified shelf to a position that is diagonal with respect to the place occupied by the escort. A 5-move (Fig. 3b) shifts the identified cell to the opposite position with respect to the place occupied by the escort.

A *clearing condition* is a shelf configuration from which it is possible to clear the path of the requested shelf towards

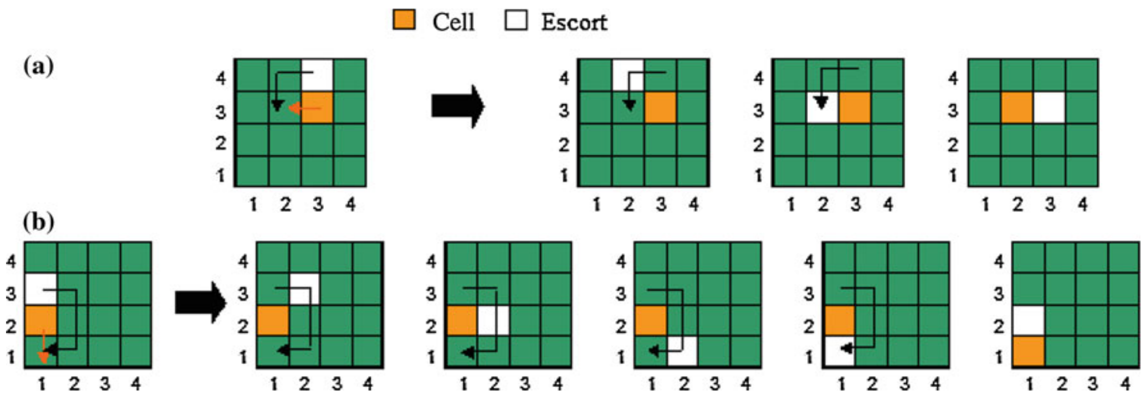


Fig. 3 **a** 3-Moves to reach the cleaning condition. **b** 5-Moves to reach the cleaning condition

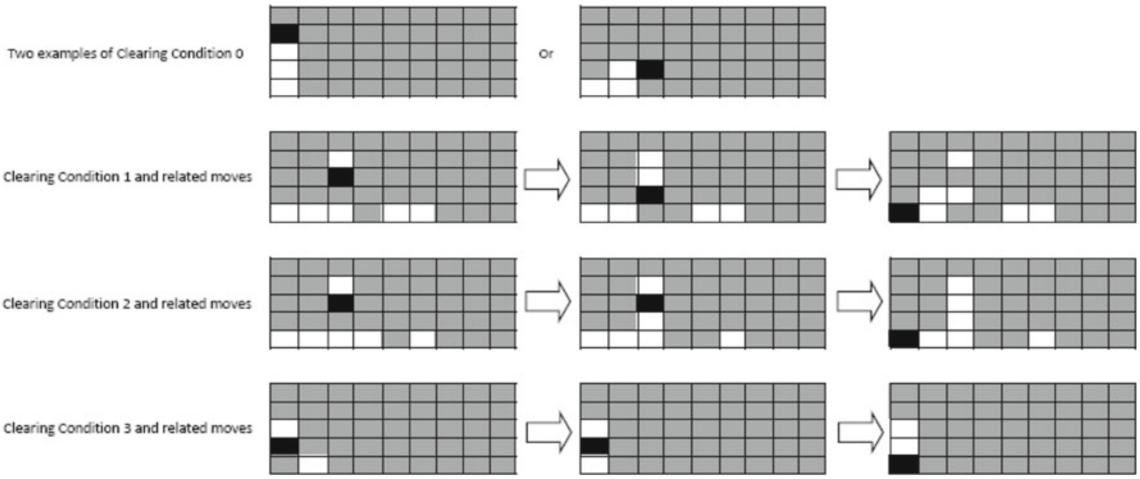


Fig. 4 “Clearing conditions” 1, 2, 3 with their resulting clearing sequences of moves

the I/O point by performing specific moves that handle the least possible number of shelves. Clearing Condition 0 is any configuration in which there are no interfering items and the requested item can be moved directly to the I/O point (Fig. 4, first line presents two examples of Clearing Conditions of type 0). Clearing Condition 1 is met if the item is in location (i′, 3), escorts occupy locations (i′, 4) and (i′, 1), and (i′+1, 1) is occupied by an item. Figure 4, second line, shows the standard moves used in this case. Clearing Condition 2 (Fig. 4, third line) is met if the item is in location (i′, 3) and escorts occupy (i′, 4), (i′, 1) and (i′+ 1, 1). Clearing Condition 3 (Fig. 4, fourth line) applies when there are two escorts.

Finally, items in Region C are “puzzled” to the aisle by using the rightmost escort as if it were the only escort present. Then, the shelf is moved to the I/O point.

AGV-powered shelf management strategies

The heuristics proposed in this paper builds on concepts similar to the ones outlined above, but with some significant differences due to the limited number of AGVs and to the potentially greater size of the warehousing system.

It is therefore possible to propose an overall breakdown of the management problem as in the flow-chart depicted in Fig. 5. A brief overview of the whole process will be given in the following, while activity 5 will be discussed in greater depth, since it is at the heart of the management problem.

In a large scale system, such as the last one that will be examined in “An application: car parking”, it is likely that a number of I/O bays will be foreseen, since a single bay could require items to travel for a long distance, ending up with unreasonably high access times and increasing likelihood of congestion and blocking (due to interference between the paths followed by shelves and AGVs). At the same time, foreseeing additional I/O bays would not lead to a substantial increase in investment and would avoid their becoming a bottleneck because of their slow operation or unreliability.

In fact, the appeal of a puzzle-based storage system using AGVs is to allow dynamic partitioning of a large warehouse (i.e., dividing it into regions that gravitate each around an I/O bay) based on the current picking list, and assigning AGVs

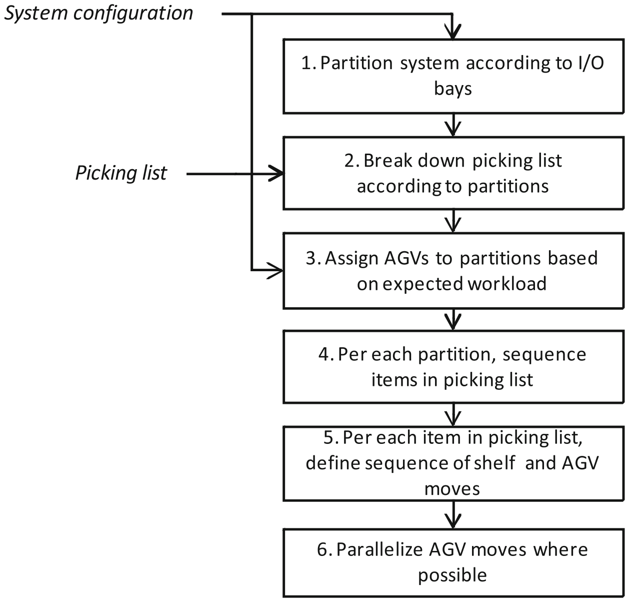


Fig. 5 Overall flow chart of AGV planning problem

to each region in order to balance the workload that the list places on each region.

So, the system can effectively be decomposed in a number of sub-warehouses, each one reliant on one I/O location. In a regularly-shaped warehouse each division can be thought to be equal in size, but adjustments can be made in the case of irregular shapes, thus assigning a lower number of shelves to the divisions in which access might be more difficult.

Given the partitions, it is straightforward to go over the items in a picking list and then assign them to each division. An item retrieval or entry means the retrieval of the shelf that carries a stored item or that must be loaded with an entering item. So, from now on it is possible to ignore the individual item and focus on shelves, not making any difference between entry or exit cycles, and calling them generically *retrievals*.

At this point, the AGV tractors can either be pre-assigned to each division simply based on its size or dynamically, based on the expected workload. In other words if—given the current picking list—a division must deal with many items and/or with items that are located far away from the I/O bay, it may be given more AGVs than a division in which the workload is likely to be lower.

Items in the picking list must then be sequenced according to some rule of choice. In this paper First-Come-First-Served has been used for simplicity, but it is of course possible to consider other criteria, such as Earliest Due Date (in case due dates have been specified) or Shortest Processing Time (where processing time can be approximated by the distance from each item to the I/O bay). The attractiveness of SPT is that, in this kind of system, actual retrieval times are not fixed, since shelves get moved around while other shelves are being retrieved. So, it makes sense to retrieve nearest shelves first, before they get moved farther away due to other shelves' movements. Instead, the furthest ones can be dealt with later on because they will either stay close to where they are or, at the most, they might find themselves closer to the I/O bay when their turn comes.

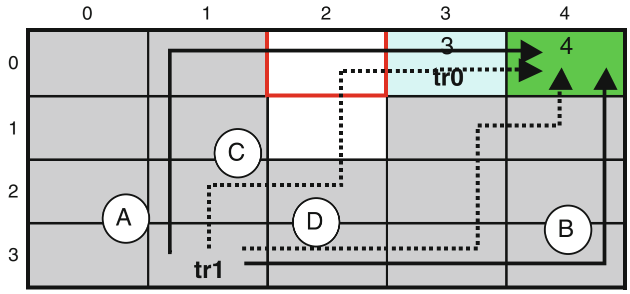


Fig. 6 Tractor 1's possible paths to shelf n.4

At this point, the major problem consists in deciding the assignment of tractors to shelves, choose the path that tractors have to follow in order to reach the assigned shelf, and suggest the direction of the steps. The objective is to minimize retrieval time (which, by the way, is correlated to other potentially interesting variables, such as energy consumption).

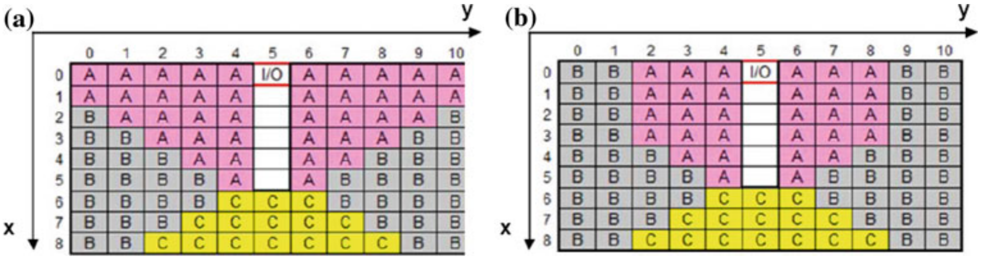
To this purpose, the storage system is modelled as a two dimension matrix with x rows and y columns, with the origin on the upper left corner; every (x, y) location is rectangular and can be either empty or occupied by a shelf. In practice, the system can have a third dimension, given by the number of levels on shelves, which can be ignored because it does not affect shelf handling and retrieval time. The I/O point has to be situated in one of the external cells of the matrix. Empty locations chosen by the user are usually more than one and are lined up at the I/O point in what we can call an *aisle*. We suppose that—following each retrieval—the aisle will be restored, while AGVs will be left at their last position.

The heuristics used shares a few insights with Gue and Kim's, but it is modified in order to consider the assignment of tractors to shelves (i.e., in which order and according to which criterion) and the choice of the path that AGVs must follow to reach the shelf assigned.

Concerning path planning for AGVs, an example is reported in Fig. 6.

In the figure there are many paths that tractor *tr1* could take to reach the green shelf in location (0, 4):

- path A, with a vertical path to (0,1) first and then a horizontal one from (0, 1) to (0, 4);
- path B, with a horizontal path to (3, 4) followed by a vertical path from (3, 4) to (0, 4);
- paths C and D, that alternate vertical and horizontal moves.



(a)

	0	1	2	3	4	5	6	7	8	9	10
0	A	A	A	A	A	I/O	A	A	A	A	A
1	A	A	A	A	A		A	A	A	A	A
2	B	A	A	A	A		A	A	A	A	B
3	B	B	A	A	A		A	A	A	B	B
4	B	B	B	A	A		A	A	B	B	B
5	B	B	B	B	A		A	B	B	B	B
6	B	B	B	B	C	C	C	B	B	B	B
7	B	B	B	C	C	C	C	C	B	B	B
8	B	B	C	C	C	C	C	C	C	B	B

(b)

	0	1	2	3	4	5	6	7	8	9	10
0	B	B	A	A	A	I/O	A	A	A	B	B
1	B	B	A	A	A		A	A	A	B	B
2	B	B	A	A	A		A	A	A	B	B
3	B	B	A	A	A		A	A	A	B	B
4	B	B	B	A	A		A	A	B	B	B
5	B	B	B	B	A		A	B	B	B	B
6	B	B	B	B	C	C	C	B	B	B	B
7	B	B	B	C	C	C	C	C	B	B	B
8	B	B	C	C	C	C	C	C	C	B	B

Fig. 7 **a** Regions in 9 × 11 grid with 6 empty locations and 5 tractors. **b** Regions in 9 × 11 grid with 6 empty locations and 3 tractors

Paths A and B warrant the smallest number of wheel rotations and can therefore be preferred. The choice between them will be made with the objective of prevent AGVs from obstructing each other and avoiding deadlocks. A number of rules can be developed to this purpose. The easiest one is to force AGVs to avoid crossing other vehicles' paths by "yielding and keeping to the right". This can be formulated according to the following rules:

- an AGV whose destination is in a straight line should have right of way over an AGV whose destination is in a diagonal, since the latter has greater flexibility in choosing its path,
- if two paths intersect, an AGV should give right of way to vehicles approaching from right,
- AGVs whose destination is placed diagonally should first move up and then right, or down and then left.

Concerning path planning of shelves, the model considers as initial condition grids with a vertical aisle and an I/O point at the top. As in the Gue-Kim heuristic, the grid is divided into three regions A, B, C, as shown in Fig. 7 for a 9 × 11 grid, though based on a different type of definition that takes the number of available tractors into account, since it is the number of tractors that allows simultaneous movement of shelves and therefore the straightforwardness of the moves to be planned. In particular, Fig. 7a reports the case with 5 tractors, while Fig. 7b the case with 3 tractors.

According to this position, shelf $s$belongs to Region A if its distance (according to the horizontal axis) from the nearest escort in the aisle is the minimum between the number of tractors N_t and the number of empty cells available between its row and the bottom on the aisle. In this case, it is straightforward to extract the shelf, because N_t-1 tractors can simultaneously move the shelves that are blocking the way to the aisle sideways and then to the dead end of the aisle. The remaining AGV is therefore free to move the required shelf sideways and then to the I/O point. This can be mathematically expressed as:

$$|y_s - y_{ne}| \leq \min\{N_t, N_e\}$$

where (x_s, y_s) represents the location of shelf s to be moved; (x_{ne}, y_{ne}) represents the escort nearest to shelf s; (x_e, y_e) represents a generic empty location; N_t is the number of available tractors; N_e is the number of empty location such that $x_e \geq x_s$.

A shelf belongs to Region C if its distance to the I/O point along the y axis is less or equal to the distance to the nearest empty location along the x axis, or:

$$|y_s - y_{io}| \leq x_s - x_{ne}$$

where (x_{io}, y_{io}) is the location of the I/O point.

All the others shelves, i.e., those not belonging to region A or C, belong to Region B.

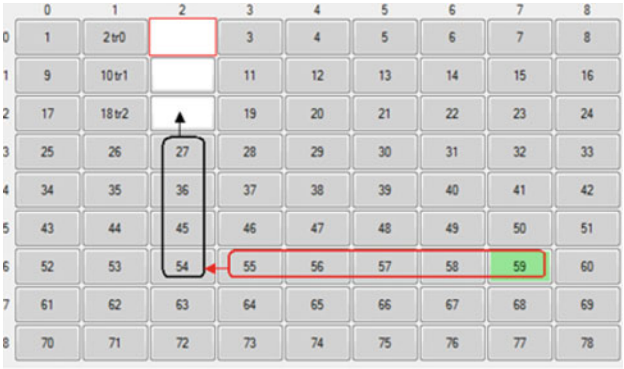
As mentioned, a shelf in Region A is retrieved simply by moving interfering shelves sideways and down, clearing the path for the requested shelf to move directly to the I/O point.

Shelves in Region B can be retrieved in two steps. First, it is necessary to move an escort close to the shelf, so as to allow its movement. In order to do this it is possible to follow the rules summarized in Table 1.

For instance, in Fig. 8, for the shelf n. 60 be retrieved and an escort to be placed on position 59, shelves 27, 36, 45 and 54 must be moved up and shelves 55–59 leftwards.

Table 1 Rules for moving the closest empty cell to the shelf and its first move direction (region B)

Aisle	Position of I/O point respect to the shelf	The escort must be brought	Direction of first move of the shelf
Vertical	Left	To the left of the requested shelf	Left
	Right	To the right of the requested shelf	Right
Horizontal	Above	Above the requested shelf	Up
	Below	Below the requested shelf	Down



	0	1	2	3	4	5	6	7	8
0	1	2tr0		3	4	5	6	7	8
1	9	10tr1		11	12	13	14	15	16
2	17	18tr2		19	20	21	22	23	24
3	25	26	27	28	29	30	31	32	33
4	34	35	36	37	38	39	40	41	42
5	43	44	45	46	47	48	49	50	51
6	52	53	54	55	56	57	58	59	60
7	61	62	63	64	65	66	67	68	69
8	70	71	72	73	74	75	76	77	78

Fig. 8 First leftward move of shelf n.59 (Region B—Step 1, view from the simulator)

Finally, a sequence of 3-moves and 5-moves is executed until a Clearing Condition is reached. The requested shelf is then extracted and the aisle rebuilt. Retrieval of a shelf in Region C is similar to the case of region B, though simpler. The first move is performed according to rules showed in Table 2, followed by 3-moves and 5-moves until Clearing Conditions are reached.

Tables 3 and 4 summarize tractors rules developed for the three regions, when the number of tractors in the system is equal to or greater than five.

A computer code that implements the procedure has been implemented in order to allow performance assessment of the storage system. The user can configure the storage system (i.e., by selecting matrix dimensions, number of tractors, location of the I/O point and number and location of empty cells) and then simulate a sequence of item retrievals or entries in order to evaluate performance indicators, which include the degree of surface exploitation achieved by the analyzed warehouse, the minimum, average and maximum times needed to retrieve a particular shelf or to perform a sequence of retrievals, the distance covered by tractors, the number of hooking or unhooking events (which provides the basis for evaluating energy consumption).

An application: car parking

A puzzle-based storage system with AGVs could be considered to be a promising answer to the problem of car parking. Traditional parking areas generally do not allow an efficient use of space, because of the areas needed for maneuvering and for entrance and exit passageways (as a rule of thumb, traditional parking lots require three times the actual space taken by cars). A puzzle-based parking lot would allow a significant increase in storage density, full automation, but would not require the significant investment attached to current automated carousel-based parking systems.

The management strategies developed in the previous section have been tested to evaluate the performance of a car parking designed as puzzle-based storage system. This has been compared to the performance of a state-of-the art high density parking system, whose conceptual design can nowadays be considered to be one that bears the closest resemblance to puzzle-based systems. In order to provide a realistic comparison with high-density systems being actually used, the following real parking systems have been considered:

1. a small-scale private system built in 2007 for 35 vehicles, with a single I/O point (Project report 03/2007);
2. a medium capacity system developed in 2008 for 84 cars, with two I/O points, one for entering and one for exiting (Project report 04/2008);
3. a high capacity underground public parking system, realized in 2008, for 284 parking spaces, based on 4 identical modules, each with an I/O point (Project report 05/2008).

Such high-density systems are multilevel and have a vertical lift that moves in a fixed corridor. The lift has a platform mounted on a telescopic arm that slides under the car in order to retrieve it and carry it to the exit (or, vice-versa, to the storage destination). The operation is straightforward for systems with a single row of shelves on each side of each corridor, since all shelves directly face the corridor. In order to allow a higher storage density, it is also possible to have two rows on each side of a corridor, with a front row facing the corridor and a back row behind it. In such case, a few slots are left vacant. Whenever an item in a back row has to be retrieved, the item in front of it can be shifted to the empty cell, thus allowing the telescopic arm to reach all the way to the back row. Though this bears some resemblance to the concept of escorts, the system is markedly different from

Table 2 Rules for moving the closest empty cell to the shelf and its first move direction (region C)

Aisle	Position of the I/O point respect to the shelf	The escort must be brought	Direction of first move of the shelf
Vertical	Above	Above the requested shelf	Up
	Below	Below the requested shelf	Down
Horizontal	Left	On the left of the requested shelf	Left
	Right	On the right of the requested shelf	Right

Table 3 Tractors rules for region A and B (if the number of tractors is greater than or equal to 5)

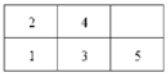
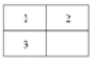
VERTICAL AISLE - I/O POINT AT THE TOP - SHELF ON THE RIGHT OF THE AISLE			
Region and steps	**Tractor-shelf assignation**		**Bringing tractors to shelves: their first move direction**
	Assignation order	**Assignation criterion**	
REGION A	Shelves handling order	Nearest tractor	Horizontal
REGION B			
(1) Black and red shelves			
black shelves (no tractors reuse)	Shelves handling order	Nearest tractor	Vertical
black shelves (tractors reuse)	Shelves handling order	First free tractor	Vertical
red shelves (n. of tractors > n. of black shelves)	Shelves handling order	Nearest tractor	Horizontal
red shelves (tractors used for black shelves)	Shelves handling order	First free tractor	Horizontal
red shelves (tractors reuse)	Shelves handling order	First free tractor	Horizontal
(2) 3-moves	***The tractor under the requested shelf stays assigned to it during the retrieval***		
first and second 3-moves	Shelves handling order	Nearest tractor	Vertical
following 3-moves (upward)	Reverse shelves handling order	Nearest tractor	Vertical
following 3-moves(leftward)	Reverse shelves handling order	Nearest tractor	Horizontal
5-moves	***The tractor under the requested shelf does not always stay assigned to it during the retrieval***		
first 5-move	Shelves handling order	Nearest tractor	Vertical
5-moves (if the previous is a 5-move too)		Nearest tractor	Vertical
5-moves (if the previous is a 3-move)		Nearest tractor	Vertical
Clearing Conditions			
Clearing Condition 0	-	-	-
Clearing Condition 1 (classical resolution)		Nearest tractor	Vertical and horizontal
Clearing Condition 1 (2b-like resolution)	Shelves handling order	Nearest tractor	Horizontal
Clearing Condition 2b	Only one shelf to move	Nearest tractor	Horizontal
Clearing Condition 2c	Only one shelf to move	Nearest tractor	Vertical
Clearing Condition 2	Only one shelf to move	Nearest tractor	Horizontal
Clearing Condition 3	Only one shelf to move	Nearest tractor	Horizontal

Table 4 Tractors rules for region C (if the number of tractors is greater than or equal to 5)

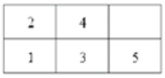
VERTICAL AISLE - I/O POINT AT THE TOP - SHELF ON THE RIGHT OF THE AISLE			
Region and steps	**Tractor-shelf assignation**		**Bringing tractors to shelves: their first move direction**
	Assignation order	**Assignation criterion**	
REGION C			
(1) Black and red shelves			
red shelves (no tractors reuse)	Shelves handling order	Nearest tractor	Horizontal
red shelves (tractors reuse)	Shelves handling order	First free tractor	Horizontal
black shelves (n. of tractors > n. red shelves)	Shelves handling order	Nearest tractor	Vertical
black shelves (tractors used for red shelves)	Shelves handling order	First free tractor	Vertical
black shelves (tractors reuse)	Shelves handling order	First free tractor	Vertical
(2) 3-moves	***The tractor under the requested shelf stays assigned to it during the retrieval***		
first and second 3-moves	Shelves handling order	Nearest tractor	Horizontal
following 3-moves (upward)	Reverse shelves handling order	Nearest tractor	Vertical
following 3-moves(leftward)	Reverse shelves handling order	Nearest tractor	Horizzontal
Mosse da 5	***The tractor under the requested shelf does not always stay assigned to it during the retrieval***		
first 5-move	Shelves handling order	Nearest tractor	Horizontal
5-moves (if the previous is a 5-move too)		Nearest tractor	Horizontal
5-moves (if the previous is a 3-move)		Nearest tractor	Horizontal

Table 5 Innovative storage system technical parameters

Shelf parameters		Tractor parameters	
Shelf length (m)	4.5	Hooking and unhooking time (s)	3
Shelf width (m)	2.5	Wheel rotation time (s)	3
Number of levels on each shelf	3	Speed of hooked tractor (m/s)	0.4
Shelf height (m)	6.6	Speed of unhooked tractor (m/s)	0.8

Table 6 Features of small capacity, medium capacity and high capacity parking systems

	Small capacity	Medium capacity	High capacity
Number of parking spaces	35	84	284
Parking levels	3	10	4
I/O points	1	2	4
System length (m)	n/a	18.25	121
System width (m)	n/a	9.70	12
Area of the parking system (m^2)	270	177	1.452
Volume of the parking system (m^3)	n/a	3.717	12.633
Area per parking space	7.7	2.1	5.1
Volume per parking space	n/a	44.3	44
Minimum retrieval time (s)	18	80	88
Average retrieval time (s)	n/a	140	137
Maximal retrieval time (s)	84	205	195

a puzzle-based one, since shelves do not move, corridors are fixed, and a single lift is in operation.

For each of the three case studies we have developed a configuration of the puzzle-based system that could match its capacity. The design strategy involves looking for symmetrical configurations with respect to the I/O point, in order to maximize the number of cells belonging to Region A.

The technical parameters that characterize the components used in the puzzle-based system are listed in Table 5. Table 6 presents the technical data and the key performance indicators of the original parking systems. Clearly, the small capacity car parking has better retrieval times than the other ones. That is justified by the lower number of vehicles in the system. However as the number of vehicles in the system grows the improving performances can be appreciated. By comparing the medium and high capacity systems it is possible to observe lower retrieval times for the high system than for the medium one. Obviously, the average and maximum retrieval time must be considered and not the minimum one that is still dependent from the number of vehicles.

The puzzle-based parking system has been simulated assuming that each tractor can perform the moves reported in Table 7 along with the required time.

The requirements of the small capacity car parking are satisfied by the system represented in Fig. 9a. It can store 36 cars, the area per parking space is 5.6m^2 and the volume per parking space is 37.2m^3 as shown in Table 8, column "small capacity". The simulation of 200 retrievals leads to the times collected in Table 9a), using a number of tractors ranging from 1 to 6.

Table 7 Tractor actions

Action	Time (s)
Shift on the right or left (long side) without shelf	5.625
Shift above or below (short side) without shelf	3.125
Shift on the right or left (long side) with shelf	11.25
Shift above or below (short side) with shelf	6.25
Shelf hooking	3
Shelf releasing	3
Wheel rotation	3
Car releasing	18

The medium capacity problem can be tackled by arranging two modules as shown in Fig. 9b. Table 8, column "medium capacity" shows that, in this case, it is possible to create 84 car spaces, the area per parking space becomes 2.1 m^2 and the volume per parking space 28.3 m^3. The results of the simulation of 56 random retrieval actions, using from 2 to 4 tractors, are reported in Table 9b).

For the high capacity parking, a configuration composed by 4 modules is proposed. Each module can store 72 cars, since it contains 24 shelves with three levels as shown in Fig. 9c. Table 8, column "high capacity", shows that in this case, the area per parking space becomes 4.7 m^2 and the volume per parking space 30.9 m^3. For this case, 192 subsequent

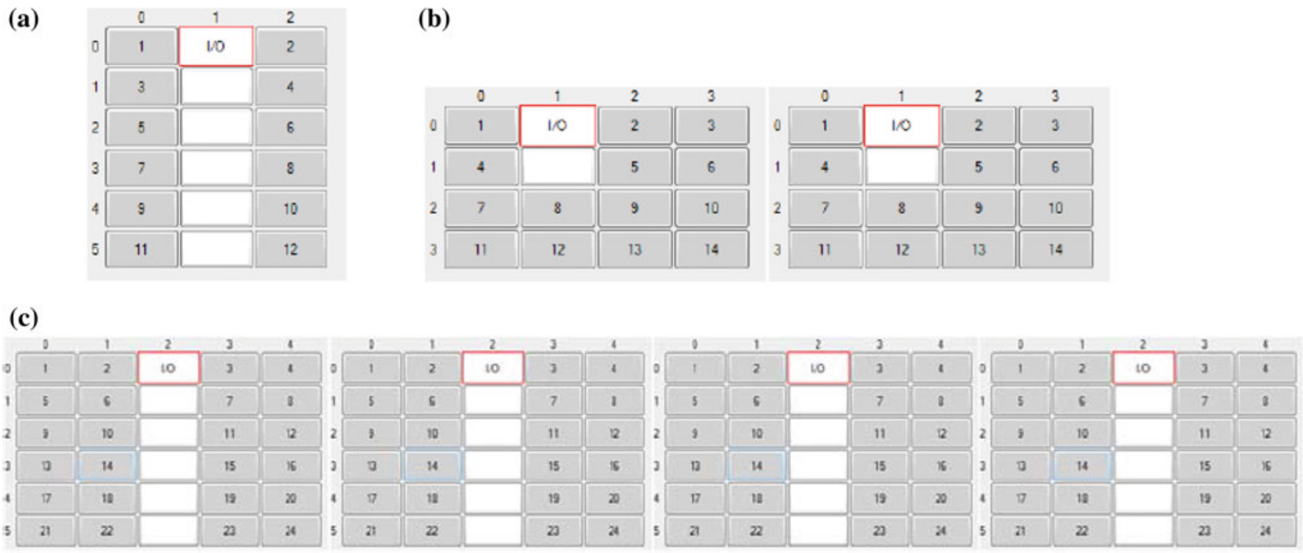


Fig. 9 Innovative system layouts storing **a** 36 cars, **b** 84 cars (42 × 2), **c** 288 cars

Table 8 Features of proposed configurations

	Small capacity	Medium capacity	High capacity
Number of parking spaces	36	84	288
Parking levels	3	3	3
I/O points	1	1	4
System length	13.5	18	90
System width	15	10	15
Area of the parking system (m^2)	202.5	180	1.350
Shelf height (m)	6.60	6.60	6.60
Volume of the parking system (m^3)	1.337	2.376	8.910
Area per parking space	5.6	2.1	4.7
Volume per parking space	37.2	28.3	30.9

random retrievals are simulated, using from 2 to 6 tractors per module and getting the results reported in Table 9c).

From Table 9, it is possible to observe that, by increasing the number of tractors, the average time perceived by customers and the maximum retrieval time are reduced for the small- and medium-sized systems. That is not necessarily true for the high capacity system, where the customer perceived time decreases while the maximum retrieval time is not necessarily reduced. This can be explained by considering that, while a high number of tractors leads to more parallel moves, it also tends to create congestion and mutual hindrances between vehicles. Finally, by comparing these performances with those of the original parking systems (reported in Table 6), it is possible to notice that the innovative system needs less space per car and also allows, in general, lower retrieval times. Again, the improvements are more appreciable for the small and medium capacity systems, while in the high capacity system the tradeoff between retrieval time reduction and congestion problems must be carefully evaluated.

Conclusion

The paper has proposed some advances in an innovative and fairly promising family of storage and retrieval systems. These puzzle-based systems allow a very high storage efficiency and good performances in access time, at the same time without requiring excessive investment in automation. With respect to previous literature, that has considered puzzle-based systems in which each shelf is motorized, the paper has proposed a cheaper alternative, in which a set of AGVs are dispatched to shelves and attend to their movements. This variant defines a more complex management scenario, since it is not only necessary to plan shelf movements but also dispatching of vehicles. A new heuristic algorithm has been proposed and shown to cope with this problem with good performance. The proposal has been validated through simulation of a sequence of retrieval, by comparing it with existing automated parking systems taken as benchmarks. The results appear to be promising and allow envisaging further

Table 9 Customer perceived retrieval times in (a) small capacity (b) medium capacity (c) high capacity proposed system configuration

Minimum time	Average time	Maximum time	Number of tractors	% Reduction average time	% Reduction maximum time
(a)					
33	63.5	90	1	–	–
33	58	84	2	−8.7	−6.7
33	56.1	84	3	−3.3	–
33	54.4	78	4	−3.0	−7.1
33	54	78	5	−0.7	–
33	53.5	78	6	−0.9	–
(b)					
33	106.2	261	2	–	–
33	89.4	216	3	−18.8	−20.8
33	86.2	201	4	−3.8	−7.5
(c)					
33	82.2	168	2	–	–
33	78.6	168	3	−4.4	–
33	73.9	171	4	−5.9	1.8
33	72.0	138	5	−2.6	−19.3
33	70.6	171	6	−1.9	23.9

research, especially in the field of shelf movement planning and control. Specifically, two application environments can be outlined. The simpler one, covered by this paper, deals with a fixed list of picking requests to be satisfied in the least time. A more complex scenario would instead deal with a dynamically changing list of picking requests, calling for a further evolution of the algorithm presented in the paper.

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